Unit-4

Propositional Logic:-

Proposition- An expression consisting of some symbols, letters and words is called a proposition or sentence or statement if it is either true or false, but not both.

ex:-

1. Delhi is capital of India.
2. $2+2=4$
3. $4>6$
4. Mumbai is in Maharashtra.
5. Do your Home work. X

Truth Value - if any proposition is TRUE then its truth value is denoted by T & if proposition is false then its truth value is denoted by F.

Example (1) ~~1<3~~ 1 is less than $3 \Rightarrow T$

(2) 14 is a odd number F.

Types of proposition -

(1) Simple Proposition - The proposition having one subject and one predicate is called a simple proposition.

ex:- (1) The flower is pink.

(2) every even no is devided by 2.

② <u>compound Proposition</u> :- Two or more simple proposition when combined by various connectivities into a single composite sentence is called compound proposition.

<u>example</u> - ① The earth is round & revolves around the sun.

② A triangle is equilateral iff all sides are equal.

<u>Logical CONNECTIVITIES</u>:- The particular words and symbols used to join two or more proposition into a single composite form of compound proposition are called logical connectives.

Logical connective words	symbol	uses
① And / conjunction / join	$\wedge$	$p \wedge q$
② or / disjunction	$\vee$	$p \vee q$.
③ Negation	$\sim$ —	$\sim p$
④ equivalent	$\leftrightarrow$	$p \leftrightarrow q$
⑤ conditional "if ... then"	$\Rightarrow$	$p \Rightarrow q$.
⑥ Biconditional if & only if (iff)	$\Leftrightarrow$	$p \Leftrightarrow q$.
⑦ NAND (NOT + AND)	$\uparrow$	$p \uparrow q$.
⑧ NOR (Not + OR)	$\downarrow$	$p \downarrow q$.
⑨ XOR	$\oplus$	$p \oplus q$.

(1) Conjunction - Any two propositions can be combined by the word And to form a compound proposition said to be the conjunction.

Truth table:-

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex- Delhi is in India & $2+2=4$ $\rightarrow$ T
Delhi is in India & $2+2=5$ $\longrightarrow$ F
Delhi is in America & $2+2=4$ F

(2) Disjunction - Any two proposition can be combined by the word OR to form a compound proposition is said to be the junction

Truth table:-

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example-
(1) Delhi is in India Or $2+2=4$ T
(2) Delhi is in India Or $2+2=5$ T
(3) Delhi is in Russia or $2+2=4$ T
(4) Delhi is in Russia or $2+2=5$ F.

Negation:- The negation proposition of any given proposition P is the proposition whose truth value is opposite of P.

Truthtable:-

P	$\sim P$
T	F
F	T

Ex:- ① $P \equiv$ The flower is pink.

② $\sim P \equiv$ The flower is not pink.

Tautologies & Contradictions- A proposition is said to be an tautologies if it contain only T in last column of truth table and if it contains only F in last column of Truth Table then it is called Contradiction.

Ex:- Show that $\{(P \vee \sim Q) \wedge (\sim P \vee \sim Q)\} \vee Q$ is tautologies.

Solution -

P	Q	$\sim P$	$\sim Q$	$(P \vee \sim Q)$ (X)	$(\sim P \vee \sim Q)$ (Y)	$X \wedge Y$	$X \wedge Y \vee Q$
T	T	F	F	T	F	F	T
T	F	F	T	T	T	T	T
F	T	T	F	F	T	F	T
F	F	T	T	T	T	T	T

~~as all the~~

As the last column only contains T, it is a tautology

Logical Equivalence :- Two propositions $P(p, q, \ldots)$ & $Q(p, q, \ldots)$ are called logically equivalent or simply equivalent or equal denoted by

$$P(p, q, \ldots) \equiv Q(p, q, \ldots),$$

If they have identical truth table.

Ex! - $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Que: Prove that ~~(p∨~q)~~

$p \vee \sim(q \wedge r)$ and $(p \vee \sim q) \vee \sim r$ are equivalent

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Laws of Algebra of proposition:

(1) Idempotent law.

(a) $p \vee p \equiv p$ & (b) $p \wedge p \equiv p$.

(2) Associative law

(a) $(p \vee q) \vee r \equiv p \vee q \vee r$ (b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(3) Commutative law.

(a) $p \vee q \equiv q \vee p$ (b) $p \wedge q = q \wedge p$

(4) Distributive law

(a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(5) Identity law

(a) $p \vee T \equiv T$ (b) $p \wedge T \equiv p$

(6) (c) $p \vee F \equiv p$ (d) $p \wedge F \equiv F$

Complement Law

(7) (a) $p \vee \neg p \equiv T$ (b) $p \wedge \neg p \equiv F$

(c) $\neg T \equiv F$ (c) $\neg F \equiv T$

(8) Involution law.

$\neg\neg p \equiv p$

(9) Demorgan's law.

(a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ (b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Conditional & Biconditional statements -

Conditional - if $p \rightarrow q$.

p only if q.

p	q	$p \rightarrow q$	$\neg p \vee q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

$\therefore p \rightarrow q \equiv \neg p \vee q$

Biconditional → p, if and only if (iff) q.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

~~Arguments~~:-

Que:- Determine whether the following is contradiction or tautogoly.

$(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

p	q	$\neg p$	$\neg q$	$B = p \vee \neg q$	$C = \neg p \vee q$	$D = \neg p \vee \neg q$
T	T	F	F	T	T	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

$A = p \vee q$	$E = A \wedge B$	$F = C \wedge D$	$E \wedge F$
T	T	F	F
T	T	F	F
T	F	T	F
F	F	T	F

all the values are F so it is a ~~taut~~ Contradiction

CONVERSE, INVERSE & CONTRAPOSITIVE of conditional statement :-

① Converse of conditional stmt -

Let $p \to q$ then $q \to p$ is converse.

Ex1 - If He is topper then he studies a lot.

If He studies then he is a topper.

② Inverse of conditional statement :-

Let $p \to q$ then $\neg p \to \neg q$ is called Inverse.

Ex2 - if X studies then X will get Knowledge.

if X donot study X will not get Knowledge

(3) <u>Contrapositive:-</u>

if $p \to q$ then $\sim q \to \sim p$ is contrapositive.

If f(x) is differentiable then it is continuous.

If f(x) is not continuous then it is not differentiable.

<u>Que</u> - Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

<u>Que</u> - $p \to (p \wedge (q \to p))$ is tautology.

<u>Que</u> - $(p \vee q \vee r) \leftrightarrow [(((p \to q) \to q) \to r) \to r]$ is tautology or not.

<u>Ans</u> Yes it is.

<u>Que</u> → $p \Rightarrow (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow R$